Module 3d: Flow in Pipes
Manning's Equation

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Manning's Equation

- Manning's Equation for velocity and flow applicable to both pipe (closed-conduit) flow and open channel flow.
- It is typically applied only in open-channel flow (fluid in contact with atmosphere).

Manning's Equation

- For Fluid Velocity in U.S. Customary Units:

$$V = \frac{1.486 R^{2/3} S^{1/2}}{n}$$

Where V = velocity (ft/sec)
R = hydraulic radius (ft)
S = slope of the energy grade line
n = Manning's roughness coefficient

Manning's Equation

- For Fluid Velocity in SI Units:

$$V = \frac{R^{2/3} S^{1/2}}{n}$$

Where V = velocity (m/sec)
R = hydraulic radius (m)
S = slope of the energy grade line
n = Manning's roughness coefficient

Manning's Equation: n Values

Surface	Best	Good	Fair	Bad
Uncoated cast-iron pipe	0.012	0.013	0.014	0.015
Coated cast-iron pipe	0.011	0.012	0.013	
Commercial wrought-iron pipe, black	0.012	0.013	0.014	0.015
Commercial wrought-iron pipe, galvanized	0.013	0.014	0.015	0.017
Smooth brass and glass pipe	0.009	0.010	0.011	0.013
Smooth lockbar and welded "OD" pipe	0.010	0.011	0.013	
Vitrified sewer pipe	0.010/0.011	0.013	0.015	0.017
Common clay drainage tile	0.011	0.012	0.014	0.017
Glazed brickwork	0.011	0.012	0.013	0.015
Brick in cement mortar; brick sewers	0.012	0.013	0.015	0.017

In: Metcalf & Eddy, Inc. (George Tchobanoglous). *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981. (Table 2-1)

Manning's Equation: n Values

Surface	Best	Good	Fair	Fair
Neat cement surfaces	0.010	0.011	0.012	0.013
Cement mortar surfaces	0.011	0.012	0.013	0.015
Concrete pipe	0.012	0.013	0.015	0.016
Wood stave pipe	0.010	0.011	0.012	0.013
Plank flumes				
Planed	0.010	0.012	0.013	0.014
Unplaned	0.011	0.013	0.014	0.015
With battens	0.012	0.015	0.016	0.016
Concrete-lined channels	0.012	0.014	0.016	0.018
Cement-rubble surface	0.017	0.020	0.025	0.030
Dry-rubble surface	0.025	0.030	0.033	0.035
Dressed-ashlar surface	0.013	0.014	0.015	0.017

In: Metcalf & Eddy, Inc. (George Tchobanoglous). *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981. (Table 2-1)

Manning's Equation: n Values

Surface	Best	Good	Fair	Bad
Semicircular metal flumes, smooth	0.011	0.012	0.013	0.015
Semicircular metal flumes, corrugated	0.0225	0.025	0.0275	0.030
Canals and ditches				
Earth, straight and uniform	0.017	0.020	0.0225	0.025
Rock cuts, smooth and uniform	0.025	0.030	0.033	0.035
Rock cuts, jagged and irregular	0.035	0.040	0.045	0.045
Dredged-earth channels	0.025	0.0275	0.030	0.033
Canals, rough stony beds, weeds on earth banks	0.025	0.030	0.035	0.040
Earth bottom, rubble sides	0.028	0.030	0.033	0.035

In: Metcalf & Eddy, Inc. (George Tchobanoglous). *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981. (Table 2-1)

Manning's Equation: n Values

Surface	Best	Good	Fair	Bad
Natural-stream channels				
Clean, straight bank, full stage, no rifts/deep pools	0.025	0.0275	0.030	0.033
Clean, straight bank, full stage, no rifts deep pools, but some weeds/stone	0.030	0.033	0.035	0.040
Winding, some pools and shoals, clean	0.033	0.035	0.040	0.045
Winding, some pools/shoals, clean, lower stages, more ineffective slope/sections	0.040	0.045	0.050	0.055
Winding, some pools/shoals, some weeds/stones	0.035	0.040	0.045	0.050
Winding, some pools/shoals, clean, lower stages, more ineffective slope/sections, stony sections	0.045	0.050	0.055	0.060
Sluggish river reaches, weedy or very deep pools	0.050	0.060	0.070	0.080
Very weedy reaches	0.075	0.100	0.125	0.150

In: Metcalf & Eddy, Inc. (George Tchobanoglous). *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981. (Table 2-1)

Manning's Equation: n Values

Nature of Surface	Manning's n Range
Concrete Pipe	0.011 – 0.013
Corrugated Metal Pipe	0.019 – 0.030
Vitrified Clay Pipe	0.012 – 0.014
Steel Pipe	0.009 – 0.011
Monolithic Concrete	0.012 – 0.017
Cement Rubble	0.017 – 0.025
Brick	0.014 – 0.017
Laminated Treated Wood	0.015 – 0.017
Open Channels	
Lined with Concrete	0.013 – 0.022
Earth, clean, after weathering	0.018 – 0.020

In: Viessman and Hammer. *Water Supply and Pollution Control, Sixth Edition*. 1998. (Table 6.1) Adapted from: *Design Charts for Open-Channel Flow*. U.S. Department of Transportation, Federal Highway Administration, Hydraulic Design Series No. 3, U.S. Government Printing Office, Washington, D.C. 1961.

Manning's Equation: n Values

Nature of Surface	Manning's n Range
Open Channels	
Earth, with grass and some weeds	0.025 – 0.030
Excavated in rock, smooth	0.035 – 0.040
Excavated in rock, jagged and irregular	0.040 – 0.045
Natural Stream Channels	0.012 – 0.017
No boulders or brush	0.028 – 0.033
Dense growth of weeds	0.035 – 0.050
Bottom of cobbles with large boulders	0.050 – 0.070
Earth, with grass and some weeds	0.025 – 0.030
Excavated in rock, smooth	0.035 – 0.040
Excavated in rock, jagged and irregular	0.040 – 0.045

In: Viessman and Hammer. *Water Supply and Pollution Control, Sixth Edition*. 1998. (Table 6.1) Adapted from: *Design Charts for Open-Channel Flow*. U.S. Department of Transportation, Federal Highway Administration, Hydraulic Design Series No. 3, U.S. Government Printing Office, Washington, D.C. 1961.

Manning's Equation

Example:

- What is the velocity of water in a 1-inch diameter pipe that has a slope of 2%, assuming that the pipe is flowing full?
  - The roughness coefficient for the pipe = n = 0.013.
  - S = 2% = 0.02
  - Calculate hydraulic radius, R.
  - R (pipe full) = D/4 = [(1 inch)/(12 in/ft)]/4 = 0.021 ft
- Substituting into Manning's Equation:

$$V = \frac{1.49}{0.013}(0.021\,ft)^{2/3}(0.02)^{1/2}$$

$$V = 114.6(0.076)(0.1414)$$

$$\mathrm{V} = 1.23\ \mathrm{ft/sec}$$

Manning's Equation

Diameter of a Pipe Flowing Full Using Manning's Equation for Velocity

$$V = \frac{1.49}{n}R^{2/3}S^{1/2}$$

$$A = \frac{\pi}{4}D^2$$

$$P = \pi D$$

$$R = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$$

Manning's Equation

Diameter of a Pipe Flowing Full Using Manning's Equation for Velocity

$$V = \frac{1.49}{n}\left(\frac{D}{4}\right)^{2/3} S^{1/2}$$

$$\frac{nV}{1.49S^{1/2}} = \left(\frac{D}{4}\right)^{2/3}$$

$$\left(\frac{nV}{1.49S^{1/2}}\right)^{3/2} = \frac{D}{4}$$

$$4\left(\frac{nV}{1.49S^{1/2}}\right)^{3/2} = D$$

Manning's Equation

- For Fluid Flow in U.S. Customary Units:

$$Q = \frac{1.486AR^{2/3}S^{1/2}}{n}$$

Where
- Q = flow (ft^3/sec)
- A = cross-sectional area of flow (ft^2)
- R = hydraulic radius (ft)
- S = slope of the energy grade line
- n = Manning's roughness coefficient

Manning's Equation

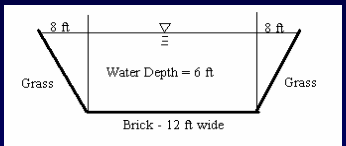
- For Fluid Flow in SI Units:

$$Q = \frac{AR^{2/3}S^{1/2}}{n}$$

Where
- Q = flow (m^3/sec)
- A = cross-sectional area of flow (m^2)
- R = hydraulic radius (m)
- S = slope of the energy grade line
- n = Manning's roughness coefficient

Manning's Equation

Example:
- Determine the discharge of a trapezoidal channel having a brick bottom and grassy sides, with the following dimensions: depth = 6 ft, bottom width = 12 ft, top width = 18 ft. Assume S = 0.002.



- For the rectangular area with the brick bottom:
  - n = 0.017

  To use Manning's, need A and R:
  - A = (Depth of Flow)(Bottom Width of Channel)
  - A = (6 ft)(12 ft) = 72 ft^2
  - P = wetted perimeter = 12 ft
  - R = A/P = 72 ft^2/12 ft = 6 ft

Manning’s Equation

- Substituting into Manning’s:

$$Q = \frac{1.49}{0.017}(72\,ft^2)(6\,ft)^{2/3}(0.002)^{1/2}$$

$$Q = 930.75\ ft^3/sec$$

$$Q = 931\ ft^3/sec$$

- For the two triangular areas with grass-lined sides:
  n = 0.025

  To use Manning’s, need A and R for one side:
  A = (0.5)(Depth of Flow)(Width of One Side)
  A = 0.5(6 ft)(3 ft) = 9 ft^2

Manning’s Equation

- To use Manning’s, need A and R for one side:
  P = hypotenuse of right triangle
  P^2 = (Depth of Flow)^2 + (Width of One Side)^2
  $P^2 = (6\text{ ft})^2 + (3\text{ ft})^2$
  $P^2 = 36\text{ ft}^2 + 9\text{ ft}^2 = 45\text{ ft}^2$
  P = 6.71 ft
  R = A/P = 9 ft2/6.71 ft) = 1.35 ft

- Substituting into Manning’s for one side of the channel:

$$Q = \frac{1.49}{0.025}(9\,ft^2)(1.35\,ft)^{2/3}(0.002)^{1/2}$$

$$Q = 29.3\ ft^3/sec$$

Manning’s Equation

Total Flow = Flow from Rectangular Section + 2(Flow from One Triangular Section)

Total Flow = 931 cfs + 2(29.3 cfs) = 989.6 cfs

Manning’s Equation

- Also can use nomographs to get solution.

From: Warren Viessman, Jr. and Mark Hammer. *Water Supply and Pollution Control Sixth Edition*. Addison-Wesley. 1998.

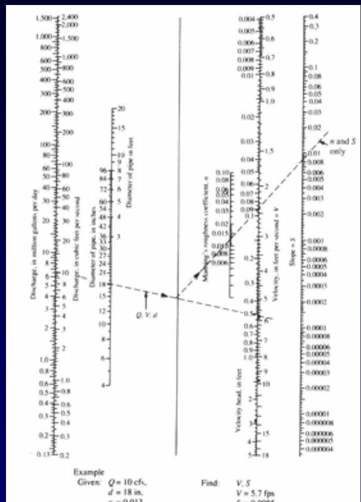


Figure 6.1 Nomograph based on Manning's formula for circular pipes flowing full.

Manning's Equation

- Also can use nomographs to get solution.

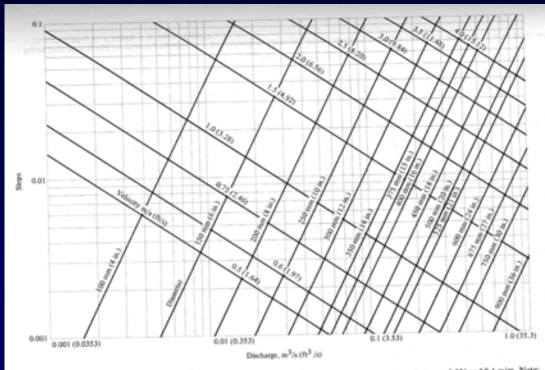


Figure 2-12 Nomograph for Manning equation ($n = 0.013$) for flows between 0.001 and 1.0 m^3/s and slopes between 0.001 and 0.1 m/m. Note: $m^3/s \times 35.3147 = ft^3/s$; $m \times 3.2808 = ft$; $mm \times 0.03937 = in.$

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

Manning's Equation

- Also can use nomographs to get solution.

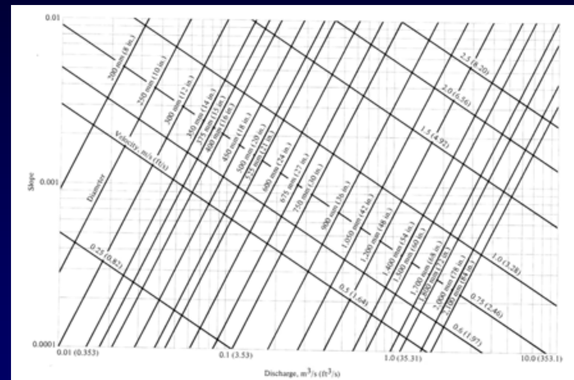


Figure 2-13 Nomograph for Manning equation ($n = 0.013$) for flows between 0.01 and 10.0 m^3/s and slopes between 0.001 and 0.01 m/m. Note: $m^3/s \times 35.3147 = ft^3/s$; $m \times 3.2808 = ft$; $mm \times 0.03937 = in.$

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

Manning's Equation

- Also can use nomographs to get solution.

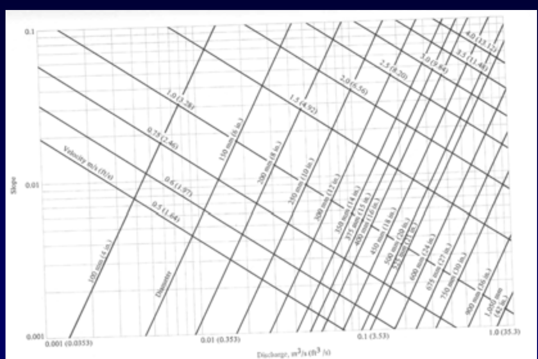


Figure 2-14 Nomograph for Manning equation ($n = 0.015$) for flow between 0.001 and 1.0 m^3/s and slopes between 0.001 and 0.1 m/m. Note: $m^3/s \times 35.3147 = ft^3/s$; $m \times 3.2808 = ft$; $mm \times 0.03937 = in.$

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

Manning's Equation

- Also can use nomographs to get solution.

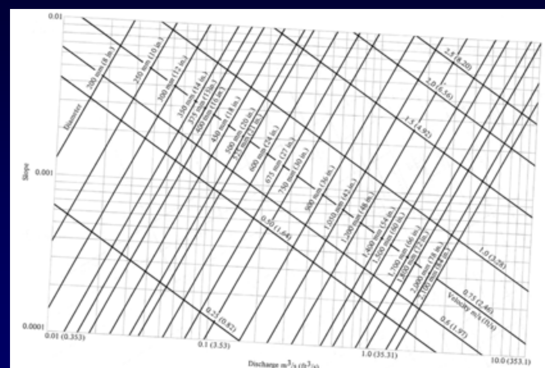


Figure 2-15 Nomograph for Manning equation ($n = 0.015$) for flows between 0.01 and 10.0 m^3/s and slopes between 0.001 and 0.01 m/m. Note: $m^3/s \times 35.3147 = ft^3/s$; $m \times 3.2808 = ft$; $mm \times 0.03937 = in.$

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

Manning's Equation

Recall that the diameter of a pipe flowing full using Manning's equation :

$$\left(\frac{4^{5/3}nQ}{1.49\pi S^{1/2}}\right)^{3/8} = D$$

Manning's Equation

Example:

- Given a 20-inch concrete conduit with a roughness coefficient of n = 0.013, S = 0.005 and a discharge capacity of 9.85 cfs, what diameter pipe is required to quadruple the capacity.
- Use Manning's equation to solve for pipe diameter:

$$D = \left(\frac{4^{5/3}nQ}{1.49\pi S^{1/2}}\right)^{3/8}$$

- To quadruple capacity, Q = 39.4 ft^3/sec.

Manning's Equation

- Substituting:

$$D = \left(\frac{4^{5/3}(0.013)(39.4\,ft^3/\sec)}{1.49\pi(0.005)^{1/2}}\right)^{3/8}$$

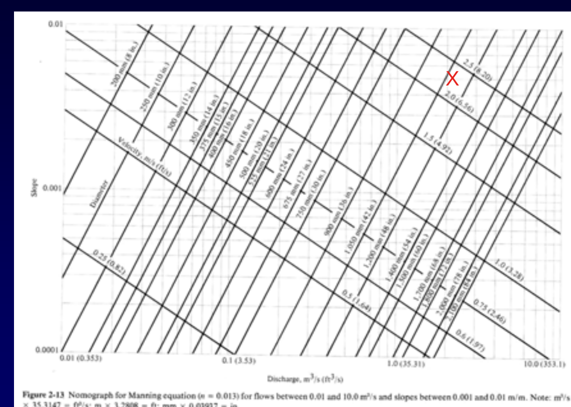
$$D = 2.8\,ft = 33.6in$$

- Want smallest commercial pipe size whose ID is greater than or equal to 33.6 in. Use a 36-in. pipe.

Or can be solved by using the nomograms.

- To use SI unit nomograms, need to convert flow rate to SI units: Q = 39.4 ft^3/sec(0.3048 m/ft)^3 = 1.116 m^3/sec = 1.12 m^3/sec

Manning's Equation



From nomogram, D should be slightly greater than 1050 mm (42 inches). Based on the nomogram, use a 48-in. diameter pipe.

Manning's Equation

Example:

- Determine the head loss in a 46-cm concrete pipe with an average velocity of 1.0 m/sec and a length of 30 m.
- For a pipe flowing full: $$R = \frac{D}{4} = \frac{0.46m}{4} = 0.115m$$
- By definition: Slope of energy line = Head Loss/Length of Pipe
  Or: Head Loss = h_L = (Slope)(Pipe Length)
- Let n = 0.012 (concrete pipe)
- Solve Manning's for Slope: $$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$S^{1/2} = \frac{nV}{R^{2/3}}$$

$$S = \left(\frac{nV}{R^{2/3}}\right)^2$$

Manning's Equation

- Substituting:

$$S = \left(\frac{0.012(1m/\sec)}{0.15^{2/3}}\right)^2$$

$$S = 0.0018$$

- Head Loss = Slope(Pipe Length)
  h_L = 0.0018(30 m) = 0.054 m
- Also can use nomogram to get the slope. For D = 460 mm and V = 1 m/sec.
  Slope = 0.0028
  Head Loss = Slope(Pipe Length)
  h_L = 0.0028(30 m) = 0.084 m
- Both solution methods show that the head loss is less than 0.10 m.

Manning's Equation

- For D = 460 mm (18 in.) and V = 1 m/sec (3.3 fps).
  Slope = 0.0028
  Head Loss = Slope(Pipe Length)
  h_L = 0.0028(30 m) = 0.084 m

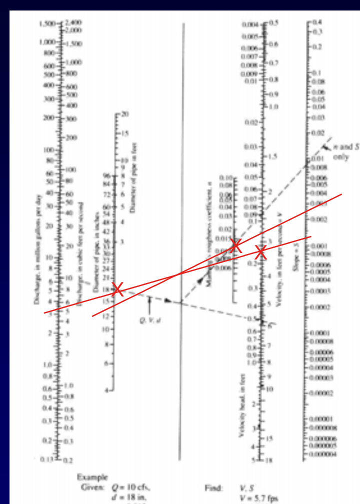


Figure 6.1 Nomograph based on Manning's formula for circular pipes flowing full.